**Are You Certain?**

**An Exploration of Epistemological Certainty in Academia**

by

Tricia Cavaliero

Submitted in Partial Fulfillment of Requirements for Honors in Psychology

Lehman College – City University of New York

Faculty Mentor: Prof. Manier

Epistemology is the study of knowledge and justified belief. It includes the *knowing* of how to do an activity, the *knowing* of a person, the *knowing* of an arbitrary object. It would be safer to say one is familiar with, than to know of, in these contexts. Conceptually saying one is familiar with X equally means they know of X, whether it be how to do X or what X is. The act of knowing, has three components. The components each answer their own question.

1. How do we know X is actually X?

2. Why do we know X?

3. How do we know we are correct in what we know about X?

To answer the first question, we must have truth; therefore, our knowledge of X needs a factual framework. The truth of something can still exist without the awareness of it, (this idea will show up later), and this idea does not suggest that unknown truths are not, in fact, true. Truth is an attribute of an idea, it exists independently. For example, everyone’s heard the saying, if a tree falls in the middle of the woods and no one is around to hear it, does it make a sound? The answer is obviously yes, however, because no one is around to hear it, can we conclude that the falling tree indeed made a sound? The sound made is independent of human hearing of the sound. Therefore, the statement is true, whether it is known to be true or not. Now, without truth we cannot then answer the second question. We know X, not only because it is true, but because we believe it to be true. For example, it’s true that Marcos and Rich are friends, but if a third person does not believe that statement, then there exists no knowledge of their friendship to that person. Truth without belief is just axiomatic; it is true, but unknowingly so. Therefore, belief must be the second element.

Now, our third question requires our first two elements. We know X is true, and we believe X is true, but why is X true? Knowledge requires an explanation. Without such, it is not knowledge, it’s belief. Taking belief further, with justification, creates the knowledge. Going back to Marcos and Rich, yes, they are friends; yes, we believe they are friends, but why do we believe? We don’t believe it because I wrote it down as a fact. There must be a separate reason or else we’ll resort back to simple belief, and my written word does not serve as a valid answer. To answer this, our third party needs justification, such as seeing them together, doing activities together, or basic examples of friendships. Once truth, belief, and justification are present, it suffices to say that knowledge is possessed.

The element of justification carries different approaches. How does one take the information given to them to then justify a proposed statement? There are two types of knowledge where they only differ in their use of valid justification. They both agree that justification covers the notion that we do not derive knowledge simply by luck. There must be factors sufficient enough to conclude our proposed statement. Justification can be either through solid full proof evidence, or through the probability that the statement is true based on logical reasoning. In theory the two types are (1) simple factual conclusions and (2) logically convincing statements. The kind focused on in this paper will be the later kind, logical convincing statements. They will be discussed in the forms of proofs, which are, generally, mathematical statements “whose truth is either to be taken as self-evident or to be assumed. Certain areas of mathematics involve choosing a set of axioms and discovering what results can be derived from them, providing proofs for the theorems that are obtained” (Clapham & Nicholson, 2014). The three types of proofs of focus for this research are of three different logical schools of thought, Formalism, Intuitionism, and Platonism.

The type of mathematical formalism used in this research is Hilbertian formalism. The goal of Hilbert’s program was to prove that all mathematics can be represented by axiomatic statements. He sought out to show that mathematics, as infinite as it is, can be written using finitary synthetic propositions. Gottlob Frege, a Platonist, critiqued formalism and said, “for the formalist, arithmetic is a game with signs which are called empty. That means that they have no other content [in the calculating game] than they are assigned by their behavior with respect to certain rules of combination [rules of the game]”(Frege, 1903). He is saying that all formalism does is create its own rules for its own game. Hilbertian formalism is a cycle of ‘man-made’ rules, which are only in play during the man-made game created following the already made rules. Hilbert’s program was proved impossible when Godel (1931) published his incompleteness theorem.

Now, Platonist ideals stem from properties not necessarily known to humans; they exist with or without human acknowledgement. An analogy of this is as such, humans are said to only have knowledge of only a portion of all the ocean and its inhabitants. That does not mean that the other entities there do not exist. It means humans simply don’t know what it is. This carries over into math and its related concepts. Laws of nature, and natural occurrences may or may not be known to humans, and it is incorrect to say that such unknown laws do not exist. Field said, “the truth-values of our mathematical assertions depend on facts involving platonic entities that reside in a realm outside of space-time” (Field, 1989). Without human awareness, it’s impossible to conclude the non-existence of natural properties.

The intuitionist approach deals with the innate perceptions of humans. Brouwer said, “if the two-ity thus born is divested of all quality, it passes into the empty form of the common substratum of all two-ities. And it is this common substratum, this empty form, which is the basic intuition of mathematics” (Brouwer, 1951). This comes from his writing where he discusses the phenomenon of the natural numbers and their properties, such as the concept of even and odd. The notion of time and the human ability to distinguish between past and present, before and after, yesterday and today. Throughout time humans have developed a system to unanimously cipher this natural awareness. This is where the set of natural numbers, or commonly known as the counting numbers, were unearthed. The natural numbers have two distinct forms, as mentioned, even and odd. From here, even and odd numbers have observable properties that are inductively provable, as well as intuitively sound. These properties are the basis for the intuitionist proofs.

This research is an attempt to find a relationship between types of proofs and specific majors. We believed that mathematics majors would find the intuitionist proofs more convincing, the philosophy majors the formalist proofs, and the psychology students the Platonist proofs. Our reasoning was as follows. Mathematics majors may form their judgments about the correctness of proofs based on the foundation of their fundamental intuitions about natural numbers (the Intuitionist approach), since mathematicians (possibly among others) can be expected to have a secure foundation in these basic mathematical intuitions (e.g., about integers). Philosophy majors may tend to rely on a string of logical statements as the ideal basis for knowledge (which in this case corresponds with the formalist approach), because their training relies on these kinds of logical manipulations. Psychology majors may hold a more *realist* approach to mathematics (which in this case corresponds with the Platonist approach), holding that mathematical objects (like squares) have definite properties. This may be because they are accustomed to considering abstract entities (such as mental schemas) as *real*.

We also hypothesized that the more advanced the students were, the more likely they would be to be able to spot the correct proofs from the incorrect proofs, when compared with less advanced (beginner) students (defined in terms of whether they have taken five or more classes in their field of major).

Methods

This study was conducted at the City University of New York at Lehman College. Majority of the participants were students, and a handful were faculty members. There were 135 participants, 74 women, 55 men and the remaining 6 did not complete the questionnaire properly. From here, 45 were classified as Psychology students, 37 advanced and 8 beginners, 27 Philosophy students, 16 advanced, and 11 beginner, 37 Mathematics students, 21 advanced, and 16 beginners, and lastly, 24 undefined participants whose results was not used in data analysis. Data collection was in the form of questionnaires, each participant was informed about the research and that they would be looking at proofs, their participation was optional, and accepting to participate was with their consent. (IRB approval was obtained.) Each questionnaire had 6 proofs, two formalist, two Platonist, and two intuitionist. Of each type, one was correct, and the other was incorrect (did not prove what it claimed to be proving). Below each proof was a scale from 1-7 where participants were asked to circle how certain they were that the proof on that page was correct.

Results

Data analysis was completed in R, and we found evidence to partially support our hypothesis. We found that those in the advanced conditions overall were significantly more able to differentiate and scale the correct proofs as more convincing than the beginner condition with p=0.0071. Advanced math participants were significantly more convinced by the correct intuitionist and Platonist proofs than the beginner math participants with p = 0.00079 and p = 0.0457.

Means (Beginners and Advanced Combined), By Proof and Major

Those in the math advanced condition were significantly more convinced than students in the psychology advanced condition by the correct intuitionist proof with p = 0.0045. We also found that psychology majors were significantly more convinced by the incorrect intuitionist proof and the incorrect formalist proof, as compared with the philosophy majors, with p = 0.0145 and p = 0.0455. With further analysis we also saw that at the beginner level psychology majors were still significantly more convinced by the same incorrect intuitionist proof with p = 0.0077. From the analysis we can say that generally speaking, our hypothesis has some support from this data.

Discussion

Generally, we found some support for our overall hypothesis that students in different majors would be more convinced by different types of proofs, but any solid conclusions about this must await further study. Our hypotheses were supported mainly insofar as Philosophy majors proved to be particularly unconvinced by the INCORRECT Formalist proof, especially as compared to Psychology majors. We had predicted that Math majors would be more convinced than Psych majors by the CORRECT Intuitionist proof, but this was found to be true only when comparing the ADVANCED Math majors to the ADVANCED Psych majors. An UNPREDICTED finding was that Psych majors were more convinced than Phil majors by an INCORRECT Intuitionist proof.

With the understanding that different majors correspond with different approaches to knowledge, future research steps would include taking this research to find the differences within majors and fields that have a wide range of schools of thought. For example, within the mathematical field it would be a future hypothesis that not only are undergrad and beginner math students less convinced by particular proofs, it is likely that the school of math one studies in grad school or for doctoral research may affect their certainty about different approaches. More applied mathematicians work with formulas day in and day out and would easily find formalist proofs more convincing because they resemble formulas and equations. Mathematicians that focus on mathematical theories may even be more convinced by Platonist ideals because even they are still working to find new theorems and postulates. Psychologist who favor quantitative studies may also find formalist proofs more convincing or even intuitionistic proof because they are able to understand the concepts of the numbers in front of them sometimes without first analyzing the data. This skill may shape how psychologists differ in this aspect. Different fields carry different variations that shape how one is most ‘convinced’.

References

Anglin, W. S. (1994). *Mathematics: A Concise History and Philosophy*. Springer-Verlag, New York.

Brouwer, L. E. J. (1981). Lectures of Intuitionism. Cambridge University Press, New York.

Clapham, C. & Nicholson, J. (2014). *The Concise Oxford Dictionary of Mathematics, Fourth edition.* Oxford University Press, New York.

Field, H (1989). *Realism, Mathematics & Modality*. Blackwell Press, New York.

Frege, G. (1903). Über die Grundlagen der Geometrie, *Jahresbericht der Deutschen Mathematiker-Vereinigung, 12* : 319–324 (Teil I), 368–375 (Teil II).

Gödel, K. (1931). "Über Formal Unentscheidbare Sätze der Principia Mathematica und Verwandter Systeme, I." *Monatshefte für Math. u. Physik 38,* 173-198.

Hersh, R. (1997). *What is Mathematics, Really?* Oxford University Press, New York.

Ichikawa, J. and Setup, M. (2018). "The Analysis of Knowledge", *The Stanford Encyclopedia of Philosophy.* Retrieved May 1, 2019 from <https://plato.stanford.edu/archives/sum2018/entries/knowledge-analysis/>>.

Iemhoff, R. (2016). "Intuitionism in the Philosophy of Mathematics", *The Stanford Encyclopedia of* Philosophy retrieved May 1, 2019 from <https://plato.stanford.edu/archives/win2016/entries/intuitionism/>.

Krantz, S. (2007). The History and Concept of Mathematical Proof. Retrieved May 1, 2019 from https://www.researchgate.net/publication/250893464\_The\_History\_and\_Concept\_of\_Mathematical\_Proof

Linnebo, O. (2018). "Platonism in the Philosophy of Mathematics", *The Stanford Encyclopedia of Philosophy*. Retrieved May 1, 2019 from <https://plato.stanford.edu/archives/spr2018/entries/platonism-mathematics/>.

Steup, M. (2018). "Epistemology", *The Stanford Encyclopedia of Philosophy.* Retrieved May 1, 2019 from <<https://plato.stanford.edu/archives/win2018/entries/epistemology/>>.

Weir, A. (2015). "Formalism in the Philosophy of Mathematics", *The Stanford Encyclopedia of Philosophy.* Retrieved May 1, 2019 from <https://plato.stanford.edu/archives/spr2015/entries/formalism-mathematics/>.